

# 第一章 逻辑代数基础

1. 运用基本定理证明下列等式。

$$(1) AB + \bar{A}C + \bar{B}C = AB + C$$

证明：

$$\begin{aligned} AB + \bar{A}C + \bar{B}C &= AB + (\bar{A} + \bar{B})C \\ &= AB + \overline{AB}C \\ &= AB + C \end{aligned}$$

□

$$(2) BC + D + \bar{D}(\bar{B} + \bar{C})(DA + B) = B + D$$

证明：

$$\begin{aligned} BC + D + \bar{D}(\bar{B} + \bar{C})(DA + B) &= BC + D + (\bar{B} + \bar{C})(DA + B) \\ &= BC + D + \overline{BC}(DA + B) \\ &= BC + D + DA + B \\ &= B + D \end{aligned}$$

□

$$(3) ABC + \bar{A}\bar{B}\bar{C} = \overline{A\bar{B} + B\bar{C} + C\bar{A}}$$

证明：

$$\begin{aligned} ABC + \bar{A}\bar{B}\bar{C} &= (A + \bar{A})(A + \bar{B})(A + \bar{C})(B + \bar{A})(B + \bar{B})(B + \bar{C})(C + \bar{A})(C + \bar{B})(C + \bar{C}) \\ &= (A + \bar{B})(A + \bar{C})(B + \bar{A})(B + \bar{C})(C + \bar{A})(C + \bar{B}) \\ &= \overline{A\bar{B} + A\bar{C} + B\bar{A} + B\bar{C} + C\bar{A} + C\bar{B}} \\ &\stackrel{\text{冗余律}}{=} \overline{A\bar{B} + B\bar{C} + C\bar{A}} \end{aligned}$$

□

$$(4) AB + BC + CA = (A + B)(B + C)(C + A)$$

证明：

$$\begin{aligned} (A + B)(B + C)(C + A) &= ABC + ABA + ACC + ACA + BBC + BBA + BCC + BCA \\ &= ABC + AB + AC + AC + BC + BA + BC + ABC \\ &= ABC + AB + AC + BC \\ &= AB + BC + CA \end{aligned}$$

□

$$(5) \bar{A}BC + AB + A\bar{C} = BC + A\bar{C}$$

证明：

$$\begin{aligned}\bar{A}BC + AB + A\bar{C} &= B(\bar{A}C + A) + A\bar{C} \\ &= B(C + A) + A\bar{C} \\ &= BC + AB + A\bar{C} \\ &= BC + A\bar{C}\end{aligned}$$

□

$$(6) \overline{A\bar{B} + \bar{A}B} = (A + \bar{B})(\bar{A} + B)$$

证明：

$$\begin{aligned}\overline{A\bar{B} + \bar{A}B} &= \overline{A\bar{B}} \overline{\bar{A}B} \\ &= (\bar{A} + B)(A + \bar{B}) \\ &= (A + \bar{B})(\bar{A} + B)\end{aligned}$$

□

$$(7) \bar{A}\bar{B} + AB + BC = \bar{A}\bar{B} + AB + \bar{A}C$$

证明：

$$\begin{aligned}\bar{A}\bar{B} + AB + BC &= \bar{A}\bar{B} + AB + BC(A + \bar{A}) \\ &= \bar{A}\bar{B} + AB + BCA + BC\bar{A} \\ &= \bar{A}\bar{B} + AB + \bar{A}BC \\ &= \bar{A}(\bar{B} + BC) + AB \\ &= \bar{A}(\bar{B} + C) + AB \\ &= \bar{A}\bar{B} + AB + \bar{A}C\end{aligned}$$

□

2. 用逻辑代数定理化简下列逻辑函数式。

$$(1) AB + \bar{A}B\bar{C} + BC$$

$$\begin{aligned}AB + \bar{A}B\bar{C} + BC &= B(A + \bar{A}\bar{C} + C) \\ &= B(A + \bar{C} + C) \\ &= B\end{aligned}$$

$$(2) \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$$

$$\begin{aligned}\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C &= \bar{B}(\bar{A}\bar{C} + A\bar{C} + AC) \\ &= \bar{B}(\bar{C} + AC) \\ &= \bar{B}(\bar{C} + A)\end{aligned}$$

$$(3) ab(cd + \bar{c}\bar{d})$$

$$ab(cd + \bar{c}\bar{d}) = abd$$

$$(4) [x\overline{(xy)}][y\overline{(xy)}]$$

$$\begin{aligned}[x\overline{(xy)}][y\overline{(xy)}] &= xy\overline{(xy)}\overline{(xy)} \\ &= xy\overline{(xy)} \\ &= 0\end{aligned}$$

$$(5) \overline{(a+b)} \overline{(\bar{a}+\bar{b})}$$

$$\overline{(a+b)} \overline{(\bar{a}+\bar{b})} = \bar{a}\bar{b}ab$$

$$= 0$$

$$(6) \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + abc$$

$$\begin{aligned} \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + abc &= \bar{a}\bar{b} + a\bar{b}\bar{c} + abc \\ &= \bar{b}(\bar{a} + a\bar{c}) + abc \\ &= \bar{b}(\bar{a} + \bar{c}) + abc \\ &= \bar{b}\bar{a}\bar{c} + bac \end{aligned}$$

4. 用卡诺图化简下列最小项表达式.

$$G = f(a, b, c) = \sum m(1, 3, 5, 6, 7)$$

		c
	ab	
	00	0
00	0	1
01	0	1
11	1	1
10	0	1

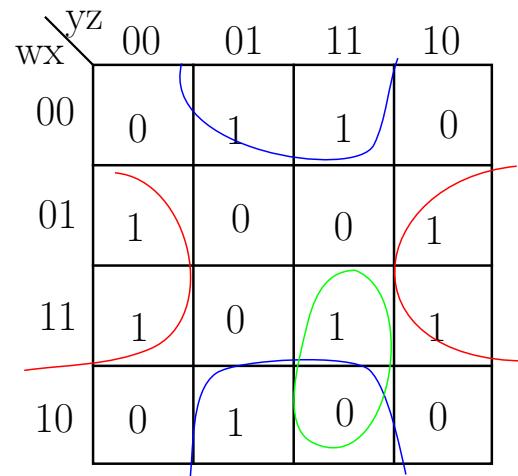
$$G = f(a, b, c) = ab + c$$

$$H = f(w, x, y, z) = \sum m(0, 2, 8, 10)$$

		yz
	wx	
	00	00
00	1	0
01	0	0
11	0	0
10	1	0

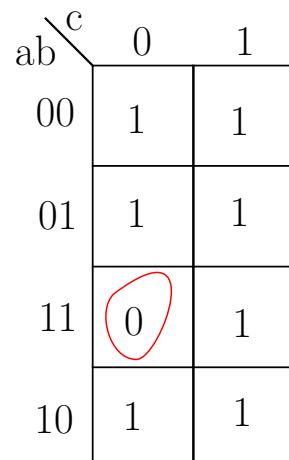
$$H = f(w, x, y, z) = \bar{x}\bar{z}$$

$$I = f(w, x, y, z) = \sum m(1, 3, 4, 6, 9, 12, 14, 15)$$



$$I = f(w, x, y, z) = x \oplus z \oplus (wyz)$$

$$J = f(a, b, c) = \sum m(0, 1, 2, 3, 4, 5, 7)$$



$$J = f(a, b, c) = \sum M(6) = \bar{a} + \bar{b} + c$$

$$K = f(a, b, c, d) = \sum m(3, 4, 5, 7, 9, 13, 14, 15)$$

ab \ cd	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	0	1	1	1
10	0	1	0	0

$$K = f(a, b, c, d) = bd + \bar{a}b\bar{c} + \bar{a}cd + abc + \bar{a}cd$$

$$L = f(a, b, c, d) = \sum m(0, 1, 2, 5, 6, 7, 8, 9, 13, 14)$$

ab \ cd	00	01	11	10
00	1	1	0	1
01	0	1	1	1
11	0	1	0	1
10	1	1	0	0

$$L = f(a, b, c, d) = \bar{b}\bar{c} + \bar{c}d + \bar{a}bc + \bar{a}cd + bc\bar{d}$$

5. 用卡诺图化简下列最大项表达式。

$$H = f(a, b, c, d) = \prod M(2, 3, 4, 6, 7, 10, 11, 12)$$

ab	cd	00	01	11	10
00	1	1	0	0	
01	0	1	0	0	
11	0	1	1	1	
10	1	1	0	0	

A Karnaugh map for four variables (a, b, c, d) with rows labeled ab (00, 01, 11, 10) and columns labeled cd (00, 01, 11, 10). The map shows the following values: (00,00)=1, (00,01)=1, (01,00)=0, (01,01)=1, (11,00)=0, (11,01)=1, (11,11)=1, (11,10)=1, (10,00)=1, (10,01)=1, (10,11)=0, (10,10)=0. Red circles highlight groups of 1s: a group of 2x2 at (00,00), a group of 2x2 at (01,01), a group of 2x2 at (11,01), and a group of 2x2 at (10,00).

$$H = f(a, b, c, d) = (a + \bar{c})(\bar{b} + c + d)(b + \bar{c})$$

$$F = f(u, v, w, x, y) = \prod M(0, 2, 8, 10, 16, 18, 24, 26)$$

uv	wx	00	01	11	10
00	y	y	1	1	
01	y	y	1	1	
11	y	y	1	1	
10	y	y	1	1	

A Karnaugh map for five variables (u, v, w, x, y) with rows labeled uv (00, 01, 11, 10) and columns labeled wx (00, 01, 11, 10). The map shows the following values: (00,00)=y, (00,01)=y, (01,00)=y, (01,01)=y, (11,00)=y, (11,01)=y, (11,11)=1, (11,10)=1, (10,00)=y, (10,01)=y, (10,11)=1, (10,10)=1. Red circles highlight groups of 2x2 at (00,00), (01,01), (11,01), and (10,00).

$$F = f(u, v, w, x, y) = w + y$$

6. 化简下列带任意项的逻辑函数。

$$V = f(a, b, c, d) = \sum m(2, 3, 4, 5, 13, 15) + \sum d(8, 9, 10, 11)$$

ab	cd	00	01	11	10
00	0	0	1	1	
01	1	1	0	0	
11	0	1	1	0	
10	d	d	d	d	

$$V = f(a, b, c, d) = ad + \bar{b}c + \bar{a}b\bar{c}$$

$$Y = f(u, v, w, x) = \sum m(1, 5, 7, 9, 13, 15) + \sum d(8, 10, 11, 14)$$

uv	wx	00	01	11	10
00	0	1	0	0	
01	0	1	1	0	
11	0	1	1	d	
10	d	1	d	d	

$$Y = f(u, v, w, x) = x\bar{u}\bar{v}w = x(u + v + \bar{w})$$

$$P = f(r, s, t, u) = \sum m(0, 2, 4, 8, 10, 14) + \sum d(5, 6, 7, 12)$$

rs \ tu	00	01	11	10
00	1	0	0	1
01	1	d	d	d
11	d	0	0	1
10	1	0	0	1

$$P = f(r, s, t, u) = \bar{u}$$

$$H = f(a, b, c, d, e) = \sum m(5, 7, 9, 12, 13, 14, 17, 19, 20, 22, 25, 27, 28, 30) + \sum d(8, 10, 24, 26)$$

ab \ cd	00	01	11	10
00	0	0	e	e
01	de	d 0	$\bar{e}$	1
11	de	de	$\bar{e}$	$\bar{e}$
10	e	e	$\bar{e}$	$\bar{e}$

$$H = f(a, b, c, d, e) = a\bar{c}e + ac\bar{e} + \bar{a}\bar{b}ce + bcd\bar{e} + b\bar{c}\bar{d} + \bar{a}bcd\bar{d}$$

$$I = f(d, e, f, g, h) = \prod M(5, 7, 8, 21, 23, 26, 30) \cdot \prod D(10, 14, 24, 28)$$

de	fg	00	01	11	10
00	1	1	$\bar{h}$	$\bar{h}$	
01	$h$	d 1	d 1	1	
11	d 1	$h$	$h$	d 1	
10	1	1	$\bar{h}$	$\bar{h}$	

$$I = f(d, e, f, g, h) = (e + \bar{f} + \bar{h})(\bar{e} + \bar{g} + h)(d + \bar{e} + f + h)$$

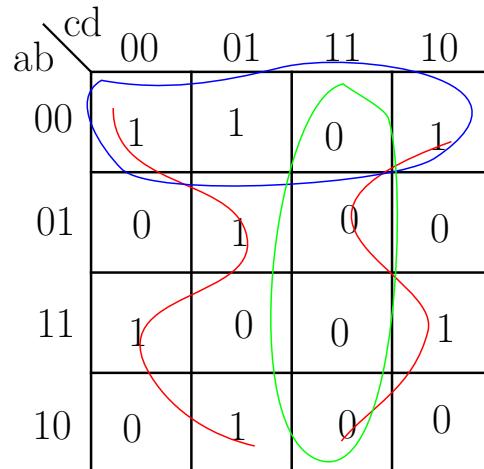
8. 将下列逻辑函数化简成与非形式最简式。

$$U = f(a, b, c, d) = \sum m(3, 4, 6, 11, 12, 14)$$

ab	cd	00	01	11	10
00	0	0	1	0	
01	1	0	0	1	
11	1	0	0	1	
10	0	0	1	0	

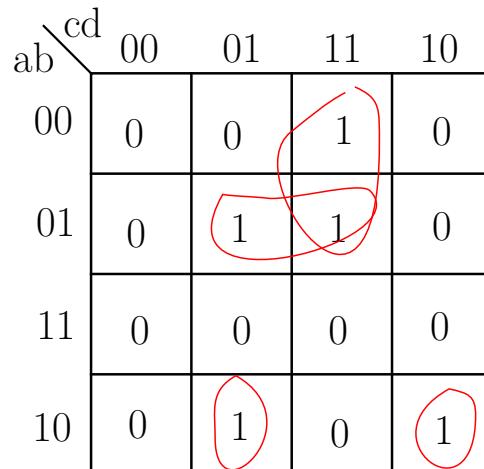
$$U = f(a, b, c, d) = b\bar{d} + \bar{b}cd = \overline{b}\overline{d} \overline{\bar{b}cd}$$

$$V = f(a, b, c, d) = \sum m(0, 1, 2, 5, 8, 10, 13)$$



$$\begin{aligned}
 V = f(a, b, c, d) &= (\overline{a \oplus b \oplus d} + \bar{a}\bar{b})\overline{cd} = \overline{a \oplus b \oplus d}\overline{\bar{a}\bar{b}cd} = \overline{(\bar{a}b + a\bar{b}) \oplus d}\overline{cd} \\
 &= \overline{(\bar{a}b + a\bar{b})d}\overline{(\bar{a}b + a\bar{b})\bar{d}}\overline{cd} = \overline{\bar{a}b}\overline{abd}\overline{\bar{a}b}\overline{abd}\overline{cd}
 \end{aligned}$$

$$W = f(a, b, c, d) = \sum m(3, 5, 7, 10, 11)$$



$$W = f(a, b, c, d) = \bar{a}cd + \bar{a}bd + a\bar{b}\bar{c}d + a\bar{b}cd\bar{d} = \overline{\bar{a}cd}\overline{\bar{a}bd}\overline{a\bar{b}\bar{c}d}\overline{a\bar{b}cd}\overline{a\bar{b}cd\bar{d}}$$

9. 将下列逻辑函数化简成或非形式最简式。

$$G = f(a, b, c, d) = \prod M(0, 1, 2, 5, 8, 10, 13)$$

ab	cd	00	01	11	10
00	0	0	1	0	
01	1	0	1	1	
11	1	0	1	1	
10	0	1	1	0	

The Karnaugh map for function G has circled minterms at (0,0), (0,1), (1,0), and (1,1). Red dashed lines connect (0,0) to (1,0) and (1,1) to (1,0). A blue dashed line connects (0,1) to (1,1).

$$G = f(a, b, c, d) = (b + d)(c + \bar{d} + a\bar{b}\bar{c}) = \overline{\overline{b + d}} \overline{\overline{c + \bar{d} + \bar{a} + b + c}}$$

$$H = f(a, b, c, d) = \prod M(3, 5, 7, 9, 11)$$

ab	cd	00	01	11	10
00	1	1	0	1	
01	1	0	0	1	
11	1	1	1	1	
10	1	0	0	1	

The Karnaugh map for function H has circled minterms at (0,0), (0,1), (1,0), and (1,1). Green dashed lines connect (0,0) to (1,0) and (1,1) to (1,0). A blue dashed line connects (0,1) to (1,1).

$$H = f(a, b, c, d) = \bar{d} + \bar{a}\bar{b}\bar{c} + ab = \bar{d} + \overline{a + b + c} + \overline{\bar{a} + \bar{b}}$$