第一章　逻辑代数基础

1. 运用基本定理证明下列等式。

(1) $AB + \bar{A}C + \bar{B}C = AB + C$

证明：

$$\begin{aligned} AB + \bar{A}C + \bar{B}C &= AB + (\bar{A} + \bar{B})C \\ &= AB + \overline{AB}C \\ &= AB + C \end{aligned}$$

□

(2) $BC + D + \bar{D}(\bar{B} + \bar{C})(DA + B) = B + D$

证明：

$$\begin{aligned} BC + D + \bar{D}(\bar{B} + \bar{C})(DA + B) &= BC + D + (\bar{B} + \bar{C})(DA + B) \\ &= BC + D + \overline{BC}(DA + B) \\ &= BC + D + DA + B \\ &= B + D \end{aligned}$$

□

(3) $ABC + \bar{A}\bar{B}\bar{C} = \overline{A\bar{B} + B\bar{C} + C\bar{A}}$

证明：

$$\begin{aligned} ABC + \bar{A}\bar{B}\bar{C} &= (A + \bar{A})(A + \bar{B})(A + \bar{C})(B + \bar{A})(B + \bar{B})(B + \bar{C})(C + \bar{A})(C + \bar{B})(C + \bar{C}) \\ &= (A + \bar{B})(A + \bar{C})(B + \bar{A})(B + \bar{C})(C + \bar{A})(C + \bar{B}) \\ &= \overline{A\bar{B} + A\bar{C} + B\bar{A} + B\bar{C} + C\bar{A} + C\bar{B}} \\ &\overset{\text{冗余律}}{=\!=} \overline{A\bar{B} + B\bar{C} + C\bar{A}} \end{aligned}$$

□

(4) $AB + BC + CA = (A + B)(B + C)(C + A)$

证明：

$$\begin{aligned} (A + B)(B + C)(C + A) &= ABC + ABA + ACC + ACA + BBC + BBA + BCC + BCA \\ &= ABC + AB + AC + AC + BC + BA + BC + ABC \\ &= ABC + AB + AC + BC \\ &= AB + BC + CA \end{aligned}$$

□

(5) $\bar{A}BC + AB + A\bar{C} = BC + A\bar{C}$

证明：

$$\begin{aligned}\bar{A}BC + AB + A\bar{C} &= B(\bar{A}C + A) + A\bar{C} \\ &= B(C + A) + A\bar{C} \\ &= BC + AB + A\bar{C} \\ &= BC + A\bar{C}\end{aligned}$$

□

(6) $\overline{A\bar{B} + \bar{A}B} = (A + \bar{B})(\bar{A} + B)$

证明：

$$\begin{aligned}\overline{A\bar{B} + \bar{A}B} &= \overline{A\bar{B}}\ \overline{\bar{A}B} \\ &= (\bar{A} + B)(A + \bar{B}) \\ &= (A + \bar{B})(\bar{A} + B)\end{aligned}$$

□

(7) $\bar{A}\bar{B} + AB + BC = \bar{A}\bar{B} + AB + \bar{A}C$

证明：

$$\begin{aligned}\bar{A}\bar{B} + AB + BC &= \bar{A}\bar{B} + AB + BC(A + \bar{A}) \\ &= \bar{A}\bar{B} + AB + BCA + BC\bar{A} \\ &= \bar{A}\bar{B} + AB + \bar{A}BC \\ &= \bar{A}(\bar{B} + BC) + AB \\ &= \bar{A}(\bar{B} + C) + AB \\ &= \bar{A}\bar{B} + AB + \bar{A}C\end{aligned}$$

□

2. 用逻辑代数定理化简下列逻辑函数式。

(1) $AB + \bar{A}B\bar{C} + BC$

$$\begin{aligned}AB + \bar{A}B\bar{C} + BC &= B(A + \bar{A}\bar{C} + C) \\ &= B(A + \bar{C} + C) \\ &= B\end{aligned}$$

(2) $\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

$$\begin{aligned}\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C &= \bar{B}(\bar{A}\bar{C} + A\bar{C} + AC) \\ &= \bar{B}(\bar{C} + AC) \\ &= \bar{B}(\bar{C} + A)\end{aligned}$$

(3) $ab(cd + \bar{c}d)$

$$ab(cd + \bar{c}d) = abd$$

(4) $[x\overline{(xy)}][y\overline{(xy)}]$

$$\begin{aligned}[x\overline{(xy)}][y\overline{(xy)}] &= xy\overline{(xy)}\ \overline{(xy)} \\ &= xy\overline{(xy)} \\ &= 0\end{aligned}$$

(5) $\overline{(a+b)}\ \overline{(\bar{a}+\bar{b})}$

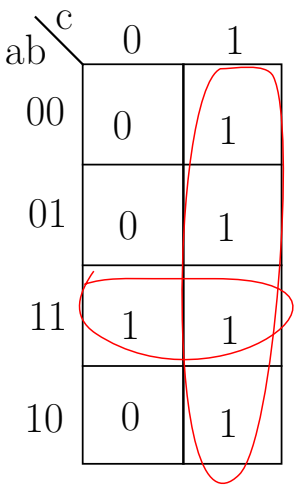
$$\overline{(a+b)}\ \overline{(\bar{a}+\bar{b})} = \bar{a}\bar{b}ab$$
$$= 0$$

(6) $\bar{a}\bar{b}\bar{c}+\bar{a}\bar{b}c+a\bar{b}\bar{c}+abc$

$$\bar{a}\bar{b}\bar{c}+\bar{a}\bar{b}c+a\bar{b}\bar{c}+abc = \bar{a}\bar{b}+a\bar{b}\bar{c}+abc$$
$$= \bar{b}(\bar{a}+a\bar{c})+abc$$
$$= \bar{b}(\bar{a}+\bar{c})+abc$$
$$= \bar{b}\overline{ac}+bac$$

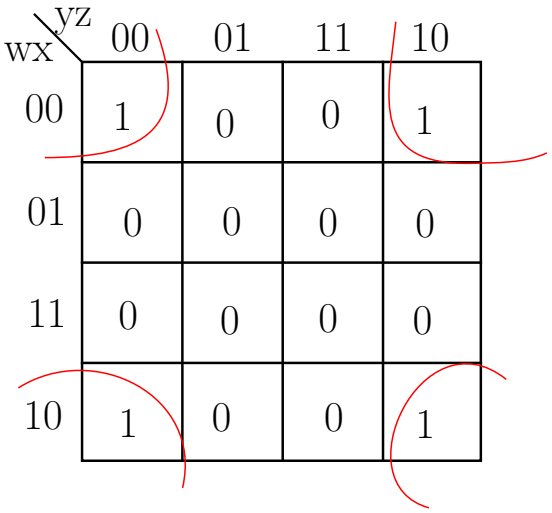
4. 用卡诺图化简下列最小项表达式.

$G = f(a,b,c) = \sum m(1,3,5,6,7)$



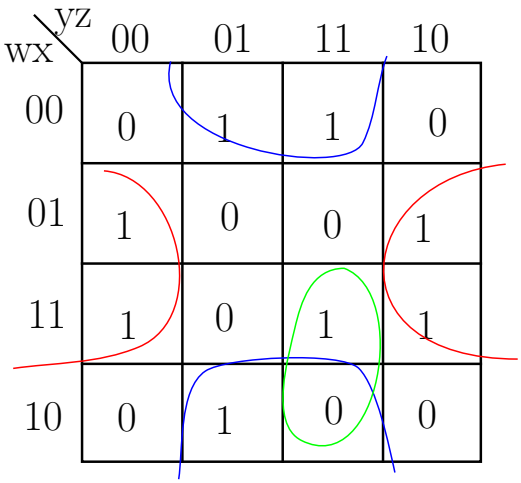
$$G = f(a,b,c) = ab+c$$

$H = f(w,x,y,z) = \sum m(0,2,8,10)$



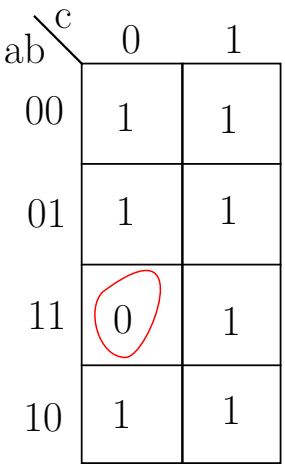
$$H = f(w,x,y,z) = \bar{x}\bar{z}$$

$I = f(w,x,y,z) = \sum m(1,3,4,6,9,12,14,15)$



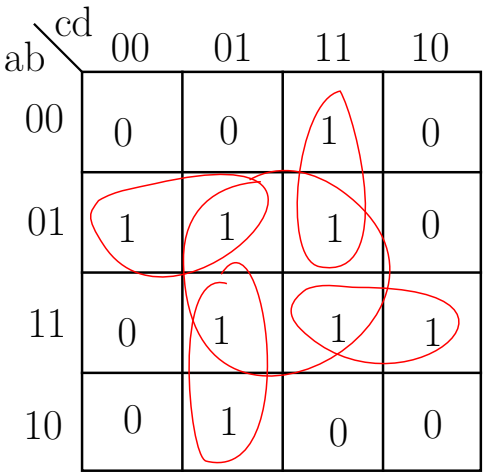
$I = f(w,x,y,z) = x \oplus z \oplus (wyz)$

$J = f(a,b,c) = \sum m(0,1,2,3,4,5,7)$



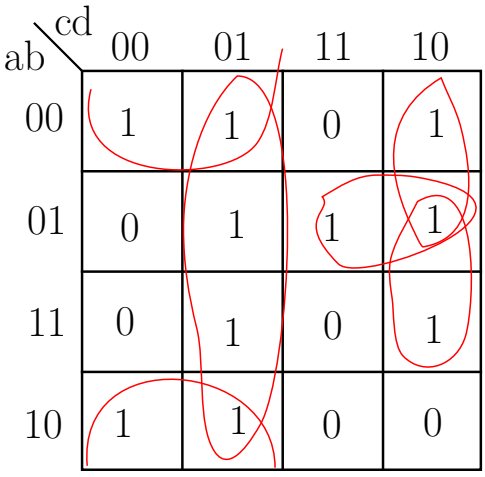
$J = f(a,b,c) = \sum M(6) = \bar{a} + \bar{b} + c$

$K = f(a,b,c,d) = \sum m(3,4,5,7,9,13,14,15)$



$$K = f(a,b,c,d) = bd + \bar{a}b\bar{c} + \bar{a}cd + abc + \bar{a}\bar{c}d$$

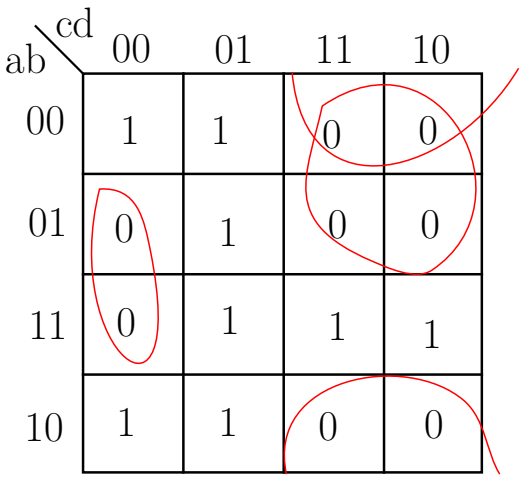
$L = f(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,13,14)$



$$L = f(a,b,c,d) = \bar{b}\bar{c} + \bar{c}d + \bar{a}bc + \bar{a}c\bar{d} + bc\bar{d}$$

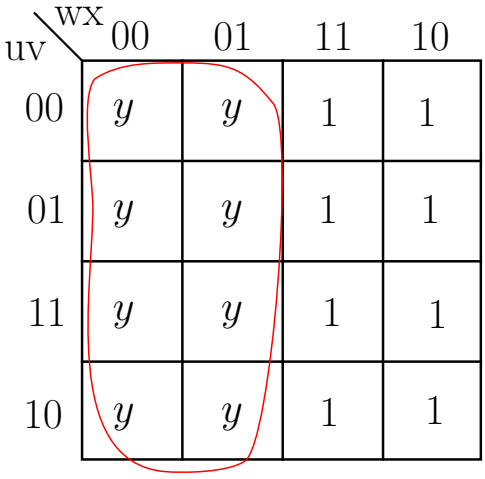
5. 用卡诺图化简下列最大项表达式。

$H = f(a,b,c,d) = \prod M(2,3,4,6,7,10,11,12)$



$$H = f(a,b,c,d) = (a+\bar{c})(\bar{b}+c+d)(b+\bar{c})$$

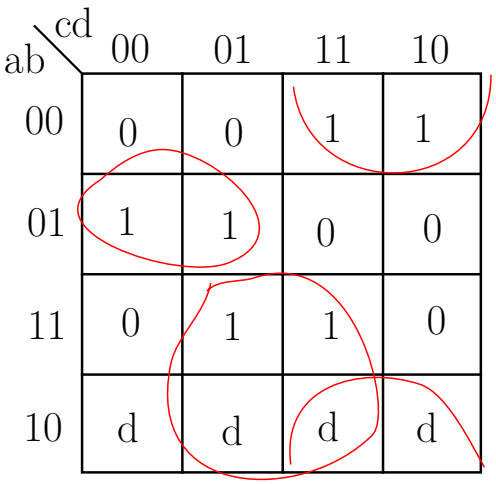
$F = f(u,v,w,x,y) = \prod M(0,2,8,10,16,18,24,26)$



$$F = f(u,v,w,x,y) = w + y$$

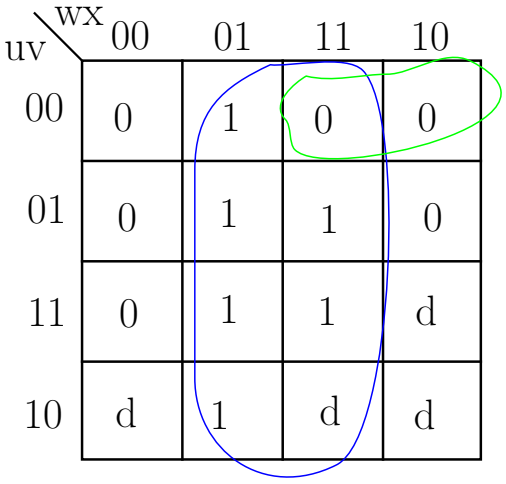
6. 化简下列带任意项的逻辑函数。

$V = f(a,b,c,d) = \sum m(2,3,4,5,13,15) + \sum d(8,9,10,11)$



$$V = f(a,b,c,d) = ad + \bar{b}c + \bar{a}b\bar{c}$$

$Y = f(u,v,w,x) = \sum m(1,5,7,9,13,15) + \sum d(8,10,11,14)$



$$Y = f(u,v,w,x) = x\overline{\bar{u}\bar{v}w} = x(u + v + \bar{w})$$

$P = f(r,s,t,u) = \sum m(0,2,4,8,10,14) + \sum d(5,6,7,12)$

rs \ tu	00	01	11	10
00	1	0	0	1
01	1	d	d	d
11	d	0	0	1
10	1	0	0	1

$P = f(r,s,t,u) = \bar{u}$

$H = f(a,b,c,d,e) = \sum m(5,7,9,12,13,14,17,19,20,22,25,27,28,30) + \sum d(8,10,24,26)$

ab \ cd	00	01	11	10
00	0	0	e	e
01	de	d 0	$\bar{e}$	1
11	de	de	$\bar{e}$	$\bar{e}$
10	e	e	$\bar{e}$	$\bar{e}$

$H = f(a,b,c,d,e) = a\bar{c}e + ac\bar{e} + \bar{a}\bar{b}ce + bcd\bar{e} + b\bar{c}\bar{d} + \bar{a}bc\bar{d}$

$I = f(d,e,f,g,h) = \prod M(5,7,8,21,23,26,30) \cdot \prod D(10,14,24,28)$

de \ fg	00	01	11	10
00	1	1	$\bar{h}$	$\bar{h}$
01	h	d 1	d 1	1
11	d 1	h	h	d 1
10	1	1	$\bar{h}$	$\bar{h}$

$$I = f(d,e,f,g,h) = (e+\bar{f}+\bar{h})(\bar{e}+\bar{g}+h)(d+\bar{e}+f+h)$$

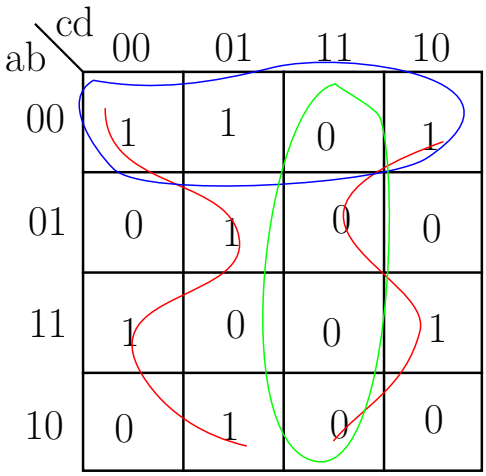
8. 将下列逻辑函数化简成与非形式最简式。

$U = f(a,b,c,d) = \sum m(3,4,6,11,12,14)$

ab \ cd	00	01	11	10
00	0	0	1	0
01	1	0	0	1
11	1	0	0	1
10	0	0	1	0

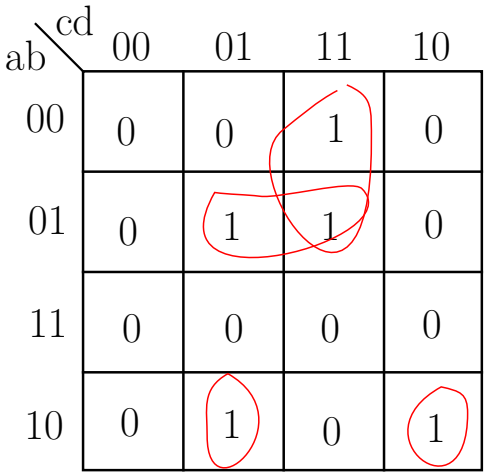
$$U = f(a,b,c,d) = b\bar{d} + \bar{b}cd = \overline{\overline{b\bar{d}}\ \overline{\bar{b}cd}}$$

$V = f(a,b,c,d) = \sum m(0,1,2,5,8,10,13)$



$$V = f(a,b,c,d) = \overline{(a \oplus b \oplus d + \bar{a}\bar{b})\bar{c}\bar{d}} = \overline{a \oplus b \oplus d}\bar{\bar{a}}\bar{\bar{b}}\bar{\bar{c}}\bar{\bar{d}} = \overline{(\bar{a}b + a\bar{b}) \oplus d}\ \bar{\bar{c}}\bar{\bar{d}}$$
$$= \overline{\overline{(\bar{a}b + a\bar{b})}d}\ \overline{(\bar{a}b + a\bar{b})}\bar{\bar{d}}\ \bar{\bar{c}}\bar{\bar{d}} = \overline{\bar{\bar{a}}\bar{\bar{b}}a\bar{\bar{b}}d}\ \overline{\bar{\bar{a}}\bar{\bar{b}}a\bar{\bar{b}}\bar{\bar{d}}}\ \bar{\bar{c}}\bar{\bar{d}}$$

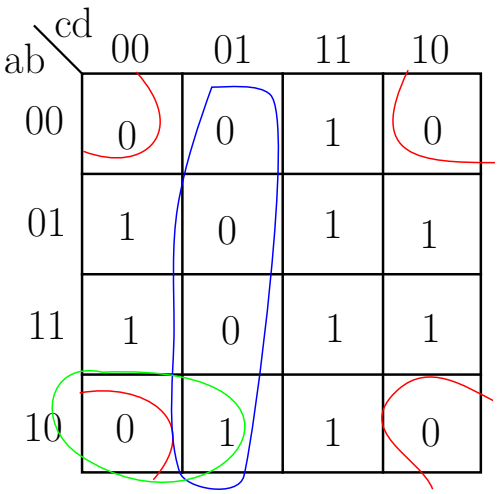
$W = f(a,b,c,d) = \sum m(3,5,7,10,11)$



$$W = f(a,b,c,d) = \bar{a}cd + \bar{a}bd + a\bar{b}\bar{c}d + a\bar{b}c\bar{d} = \overline{\bar{a}cd}\ \overline{\bar{a}bd}\ \overline{a\bar{b}\bar{c}d}\ \overline{a\bar{b}c\bar{d}}$$

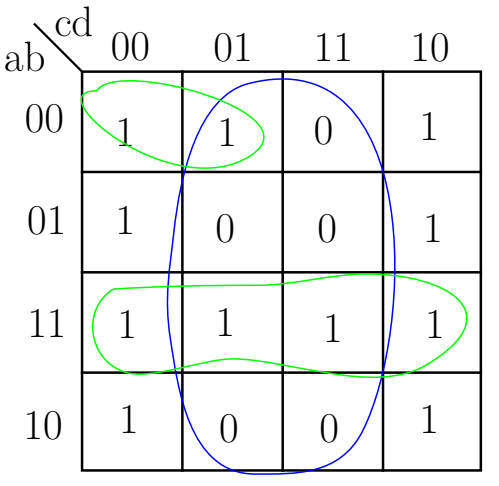
9. 将下列逻辑函数化简成或非形式最简式。

$G = f(a,b,c,d) = \prod M(0,1,2,5,8,10,13)$



$$G = f(a,b,c,d) = (b+d)(c+\bar{d}+a\bar{b}\bar{c}) = \overline{\overline{b+d}\ \overline{c+\bar{d}+\bar{a}+b+c}}$$

$H = f(a,b,c,d) = \prod M(3,5,7,9,11)$



$$H = f(a,b,c,d) = \bar{d} + \bar{a}\bar{b}\bar{c} + ab = \bar{d} + \overline{a+b+c} + \overline{\bar{a}+\bar{b}}$$